

FERUM (Finite Element Reliability Using Matlab)

J.-M. Bourinet

LaMI / IFMA - Clermont-Ferrand, France

Un bref historique

▲ Création

- ↳ Eté 1999, UCB - Créateur : T. Haukass
- ↳ Objectif pédagogique à la demande d'A. Der Kiureghian

▲ Développements à UC Berkeley

- ↳ 1999 - 2003 (UCB) - Coordinateur : T. Haukass
- ↳ Implémentation des méthodes fiabilistes usuelles (FORM, SORM, MC/IS, sensibilités) plus quelques unes plus spécifiques (fiabilité système, champs aléatoires, EFS, FORM inverse)
- ↳ Contributions via thésards, stagiaires, post-docs à UC Berkeley
- ↳ Dernière version distribuée : v 3.1 - <http://www.ce.berkeley.edu/FERUM/>

▲ Développements parallèles ...

- ↳ 2001-2002 (UCB, DGA/CTSN) et depuis 2004 (IFMA) - J.-M. Bourinet
- ↳ Reprise de la "philosophie" du code, adaptation de sa structure (vectorisation des calculs, communication avec codes externes), ajout de nouvelles fonctionnalités (sensibilités, DS, subset simulation, ²SMART, RBDO)

Program Overview

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FERUM Home

FERUM currently consists of 9 parts:

FERUMcore contains the core algorithms to perform FORM and simulation reliability analysis. This part of FERUM is maintained by Teije Hinkens.

FERUMlinearfecode is a single finite element code provided with FERUM to enable linear finite element reliability analysis with truss, beam or quad4 elements. Limit-state functions can be defined in terms of displacement response from this code. Gradients can be computed either by direct differentiation (DDM) or by forward finite difference scheme. This part of FERUM is maintained by Teije Hinkens.

FERUMnonlinearfecode is an add-on to FERUMlinearfecode to enable nonlinear finite element reliability analysis. The 2D plasticity material is provided, and gradients can be computed by direct differentiation (DDM) or by forward finite difference. Truss and quad4 elements are available. This part of FERUM is maintained by Teije Hinkens.

FERUMdynamicfecode is yet another extension of FERUMlinearfecode to enable limit-state functions being defined in terms of response quantities from a dynamic finite element analysis. This part of FERUM is maintained by Teije Hinkens.

FERUMlargedefcode is an add-on to enable limit-state functions being defined in terms of response quantities from a finite element code capable of large deformation analysis. This part of FERUM is maintained by Teije Hinkens.

FERUMsystems enables FERUM to perform system reliability analysis. This part of FERUM is maintained by [Julko Szec](#).

FERUMrandomfield is an add-on to the single finite element codes provided with FERUM. It addresses the issue of characterizing material properties as random fields. Options for the single 1D case was provided with the initial version of FERUM. However, the main contribution to the current version have been made by [Zeynep Suleci](#), who has also provided a user's theory manual for the random field part of FERUM (see the User's Guide section).

FERUMFedEx is a section enables the finite element program FedEx.exe developed by [Professor Ellis Fdypou](#) at UC Berkeley to be connected to FERUM. This provides for a quite powerful computational platform for finite element reliability analysis. This part is maintained by [Felix Fdypou](#).

FERUMexamples contains a collection of example inputfiles for FERUM.

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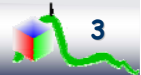
These steps will enable you to run FERUM on your computer (if you have Matlab installed):

1. Make a directory on your harddisk where you want to save the files, for example 'ferum' or 'reliability'.
2. Download and extract the parts of FERUM that you are interested in. FERUMcore is mandatory; you need this before you can do anything. Here is the list of the zip archive files available (Note that the SSEEM toolbox can be run as a stand-alone package):
 - [FERUMcore](#) with new contributions by Anthony Hinkel
 - [FERUMlinearfecode](#)
 - [FERUMdynamicfecode](#) by [Julko Szec](#)
 - [FERUMnonlinearfecode](#) by [Ernst Suleci](#)
 - [Sectional Mechanics Finite Elements in Matlab \(SSEEM\)](#) by [Ernst Suleci](#) (type SSEEM for help)
 - [FERUMdynamicfecode](#) by [Felix Fdypou](#)
 - [FERUMexamples](#)

(See the [User's Guide](#) page for more information.)

3. Before you can start using the toolbox, you need to make a 'path' in Matlab to the directory where you saved the files (so that Matlab knows where to look for them). This is done by using the 'Set Path...' option in the 'File' menu in Matlab.
4. To remove the FERUM toolbox at your disposal. Check "Getting Started" or the "User's Guide" for further help.

Please direct any general questions you may have to hankens@ce.berkeley.edu.



Principes de base

▲ Structuration des données d'entrée

↳ Variables de type structure ► stockées dans un script `inputfile_xxx.m`

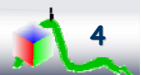
<code>probdata</code>	Données du modèle probabiliste (Nataf)
<code>analysisopt</code>	Options des méthodes de calcul
<code>gfundata</code>	Définition de la (des) fonction(s) de performance
<code>femodel</code>	Paramètres spécifiques au programme appelé par la fonction de performance
<code>randomfield</code>	Définition du champ aléatoire et de ses paramètres
<code>system</code>	Définitions de la représentation minimale par coupes

▲ Lancement de FERUM

```
>> inputfile_xxx
>> ferum
>> (numéro de l'option de calcul)
```

▲ Post-traitement

↳ Une variable de type structure spécifique à la méthode de calcul employée est créée dans l'environnement de travail. Exemple : `formresults`



Communication avec codes externes

▲ Fonction de performance gfun.m vectorisée (IFMA)

- ↳ Tous les algorithmes de calcul fiabiliste sont réécrits pour tirer bénéfice de calculs indépendants envoyés et exécutés en simultané
- ↳ Autorise un calcul Matlab vectorisé

$$R \sim N\left(7, \frac{5}{\sqrt{3}}\right) \quad S \sim N\left(2, \frac{2}{\sqrt{3}}\right) \quad g = r - s \quad MC : N = 1.5 \times 10^9$$

Non vectorisé ► 6 jours 15 h

Vectorisé ► 31 min

- ↳ Permet un calcul distribué sur machine multi-processeurs



▲ Couplages réalisés avec (IFMA) :

- ↳ Tous codes avec fichiers d'entrée paramétrés
(Gibi/gmsh + Aster, Nasgro, XFEM LaMCoS, tous codes EF commerciaux)
- ↳ Autres codes avec génération de fichiers via Matlab
(MAN/EVE LMA Marseille/LaMI)



Structure analysisopt

```
analysisopt.multi_proc = 1;  
analysisopt.block_size = 1000;
```



Structure gfundata (cas analytique)

```
% Type of limit-state function evaluator (Alternatives: 'basic', 'aster', 'fcgxfem', 'eve' )
gfundata(1).evaluator = 'basic';
gfundata(1).type = 'expression';

gfundata(1).expression = 'x1^thetag1 - thetag2*x2 + thetag3';

gfundata(1).dgdq = { ' thetag1*x1^(thetag1-1) '
                    ' -thetag2 ' };

gfundata(1).thetaname = {'thetag1', 'thetag2', 'thetag3' };
gfundata(1).thetag = [1 1 0];

gfundata(1).flag_sens = 1;
```



Structure gfundata (cas fonction Matlab)

```
% Type of limit-state function evaluator (Alternatives: 'basic', 'aster', 'fcgxfem', 'eve' )
gfundata(1).evaluator = 'basic';
gfundata(1).type = 'expression';
gfundata(1).expression = 'N_Virkler(logC,m,W,e,sigma,R,a0,af,na)-N0';

gfundata(1).cgname = { 'N0' 'W' 'e' 'sigma' 'R' 'af' 'na' };
gfundata(1).cg = [ 400000 6*25.4 0.1*25.4 23.4e3/(6*25.4*0.1*25.4) 0.2 49.8 1001 ];

gfundata(1).flag_sens = 0;
```



Structure gfundata (cas code externe)

```
% Type of limit-state function evaluator (Alternatives: 'basic', 'aster', 'fcgxfem', 'eve' )
gfundata(1).evaluator = 'aster';
gfundata(1).type = 'expression';

gfundata(1).expression = 'u0 - gext';

gfundata(1).flag_sens = 0;
```



Structure femodel (couplage Gibi + Aster)

```
femodel.data = [ 0    1  nan ;
                 0    2  nan ;
                 0    3  nan ;
                 0    4  nan ;
                 0    5  nan ;
                 0    6  nan ;
                 2    7  nan ;
                 0    8  nan ;
                 1    9  nan ;
                 1   10  nan ;
                 2   11  nan ];

femodel.auxva = 1;

% Computer platform parameters - Enter paths with slashes (not backslashes), even for windows
femodel.mesh_generator      = 'dgib';
femodel.mesh_generator_arg  = '';
femodel.jobdir_name        = 'exec';
femodel.job_name           = 'tuyau';
femodel.keep_optional_files = 0; % 1: keep all files, 0: erase files
```



Structure femodel (couplage Aster sous Windows)

```
if strcmp(analysisopt.client_OS,'win')
    femodel.code_pathname      = 'C:/aster';
    femodel.script_path       = 'C:/temp/aster/seq2/scripts';
    femodel.template_path     = 'C:/temp/aster/seq2/templates';
    femodel.work_parent       = 'C:/temp/aster/seq2/exeexecdir';
    femodel.client_dir        = 'C:/temp/aster/temp2';
    femodel.gawk_path_name    = 'C:/gawk/gawk.exe';
```



Structure femodel (couplage Aster sous Linux)

```
else
    femodel.code_pathname      = '/opt/aster';
    femodel.script_path       = '/home/prof/bourinet/aster/distrib2/scripts';
    femodel.template_path     = '/home/prof/bourinet/aster/distrib2/templates';
    femodel.work_parent       = '/home/prof/bourinet/aster/distrib2/exeexecdir';
    femodel.queue_name        = 'q8h';
    femodel.user_IP           = 'bourinet@172.16.40.1';
    if strcmp(analysisopt.client_OS,'linux')
        femodel.client_dir    = '/home/prof/bourinet/aster/temp2';
    elseif strcmp(analysisopt.client_OS,'master')
        femodel.client_dir    = '/home/prof/bourinet/aster/temp2';
    elseif strcmp(analysisopt.client_OS,'winftp')
        femodel.client_dir    = 'C:/temp/aster/temp2';
        femodel.puttydir      = 'D:/Putty';
        femodel.ftppasswd     = '*****';
        femodel.putty_profile = 'bourinet';
    elseif strcmp(analysisopt.client_OS,'winsamba')
        femodel.client_dir    = 'C:/temp/aster/temp2';
        femodel.work_client   = 'Z:/aster/distrib2/exeexecdir';
        femodel.puttydir      = 'D:/Putty';
        femodel.putty_profile = 'bourinet';
    end
end
```



Transformation isoprobabiliste

▲ Modèle de Nataf (UCB & IFMA)

Liu & Der Kiureghian, 1986

$$\underline{X} \text{ avec } \begin{matrix} f_{X_i}, F_{X_i} \\ \underline{R} = [\rho_{ij}]_{n \times n} \end{matrix} \quad \blacktriangleright \quad \underline{Z} \sim N(\underline{0}, \underline{R}_0) \quad \blacktriangleright \quad \underline{U} \sim N(\underline{0}, \underline{I})$$

Passage à des lois normales corrélées

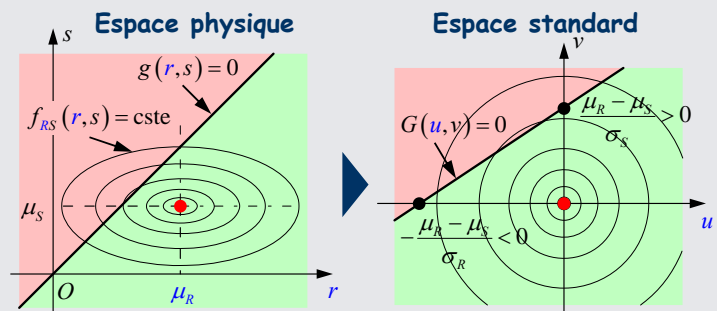
$$\Phi(z_i) = F_{X_i}(x_i)$$

$$\underline{u} = \underline{L}_0^{-1} \underline{z}$$

Décorrélation

Matrice de corrélation \underline{R}_0 calculée :

- ↳ de manière analytique par formules approchées (UCB)
- ↳ par intégration numérique (IFMA)



Lois disponibles

- | | | |
|---------------------------|-----------------|------------------------------|
| ↳ Normale | ↳ Chi deux | ↳ Gumbel (Type I, E1-max) |
| ↳ Normale tronquée (IFMA) | ↳ Exponentielle | ↳ Gumbel (Type I, E1-min) |
| ↳ Lognormale | ↳ Gamma | ↳ Frechet (Type II, E2-max) |
| ↳ Uniforme | ↳ Rayleigh | ↳ Weibull (Type III, E3-min) |
| | ↳ Beta | |



Structure probdata

```

probddata.name = { 'X1'
                  'X2'
                  'X3' };

probddata.marg = [ 2   500  100   500 nan  nan  nan   nan 0;
                  2  2000  400   2000 nan  nan  nan   nan 0;
                  6    5   0.5     5 nan  nan  nan   nan 0];

probddata.correlation = [ 1.0  0.3  0.2 ;
                          0.3  1.0 -0.2 ;
                          0.2 -0.2  1.0 ];

probddata.transf_type = 3;
probddata.Ro_method = 1;
probddata.flag_sens = 1;
    
```



First-Order Reliability Method (FORM)

$$G(\underline{u}) \underset{=0}{\approx} G(\underline{u}^*) + \nabla_{\underline{u}} G(\underline{u}^*)^T (\underline{u} - \underline{u}^*) = \underline{G}_1(\underline{u})$$

$$\underline{u}^* = \underset{\underline{u}}{\operatorname{argmin}} \{ \|\underline{u}\| \mid G(\underline{u}) = 0 \}$$

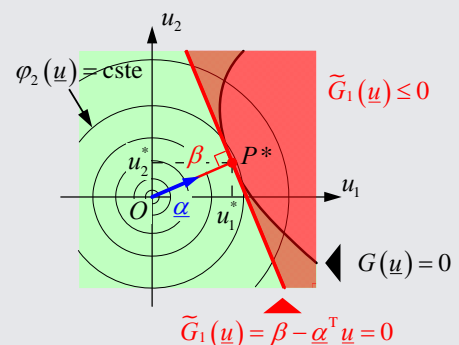
Algorithm i-HLRF (UCB & IFMA)

Liu & Der Kiureghian, 1991
Zhang & Der Kiureghian, 1997

Gradients évalués par :

- ↳ différences finies - Perturbation unique (UCB), choix par variable aléatoire (IFMA)
- ↳ différentiation directe - Direct Differentiation Method (DDM)

Possibilité de stockage de tous les paramètres à chaque itération (IFMA)



Structure analysisopt

```
% Maximum number of iterations allowed in the search algorithm
analysisopt.i_max = 500;

% Tolerance on how close design point is to limit-state surface
analysisopt.e1 = 0.001;

% Tolerance on how accurately the gradient points towards the origin
analysisopt.e2 = 0.001;

% 0: step size by Armijo rule, otherwise: given value is the step size.
analysisopt.step_code = 0;

analysisopt.Recorded_u = 1;
analysisopt.Recorded_x = 1;

% 'ddm': direct differentiation, 'ffd': forward finite difference
analysisopt.grad_flag = 'ffd';

% Parameter for computation of FFD gradients - Perturbation = stdv/analysisopt.ffdpara;
% Recommended values: 1000 for basic limit-state functions, 50 for FE-based limit-state functions
analysisopt.ffdpara = 1000;
```



Test

▲ Variables aléatoires (2)

$$X_1 \sim N(0,1) \quad X_2 \sim N(0,1)$$

▲ Fonction de performance g

$$g(x_1, x_2) =$$

▲ Probabilité de défaillance

$$p_f = 1.83 \times 10^{-3} \quad (\text{FORM} - 75 \text{ appels à } g)$$

$$p_f = 1.95 \times 10^{-3} \quad (\text{SORM-cf} - 5 \text{ appels à } g)$$

$$p_f = 1.97 \times 10^{-3} \quad (\text{SORM-pf} - 10 \text{ appels à } g)$$

$$p_f = 1.93 \times 10^{-3} \quad (\text{IS} - 1000 \text{ appels à } g)$$



Test

▲ Variables aléatoires (2)

$$X_1 \sim N(0,1) \quad X_2 \sim N(0,1)$$

▲ Fonction de performance g

$$g(x_1, x_2) = b - x_2 - \mu(x_1 - e)^2$$

with: $b = 5$, $\mu = 0.5$, $e = 0.1$

▲ Probabilité de défaillance

$$p_f = 1.83 \times 10^{-3} \quad (\text{FORM - 75 appels à } g)$$

$$p_f = 1.95 \times 10^{-3} \quad (\text{SORM-cf - 5 appels à } g)$$

$$p_f = 1.97 \times 10^{-3} \quad (\text{SORM-pf - 10 appels à } g)$$

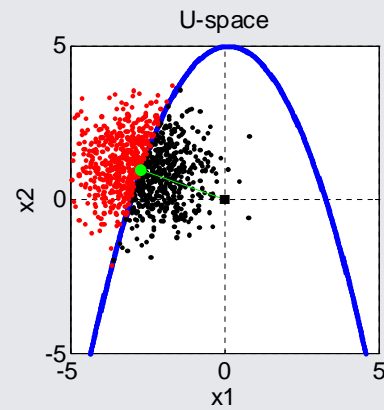
$$p_f = 1.93 \times 10^{-3} \quad (\text{IS - 1000 appels à } g)$$

$$p_f = 3.016 \times 10^{-3}$$

(DS, 100 dir)

$$p_f = 3.017 \times 10^{-3}$$

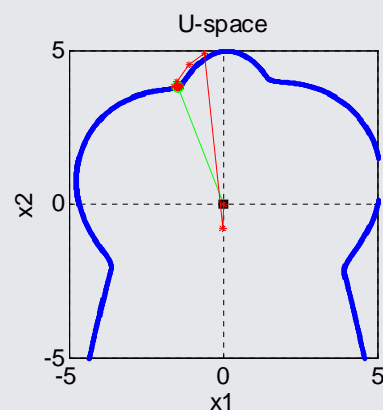
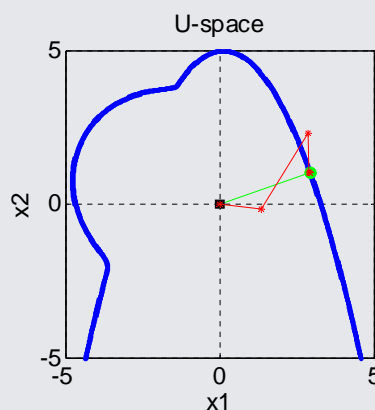
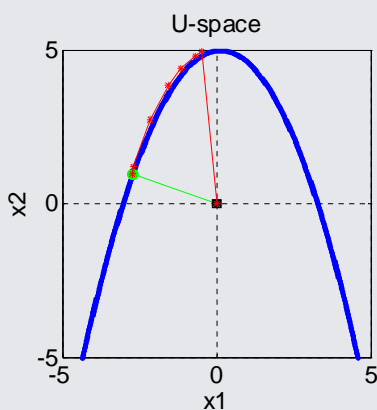
(SS, 3×10^5 sim/pas, 500 répl.)



Multi-FORM

▲ FORM pour points de conception multiples (IFMA)

Der Kiureghian & Dakessian, 1998

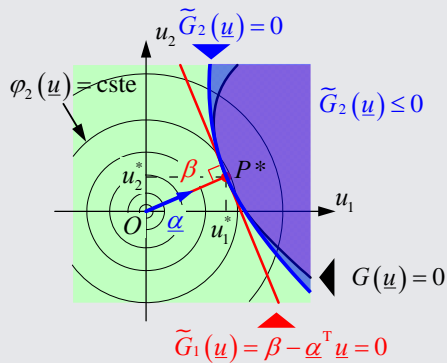


Second-Order Reliability Method (SORM)

$$G(\underline{u}) \approx \underbrace{G(\underline{u}^*)}_{=0} + \nabla_{\underline{u}} G(\underline{u}^*)^T (\underline{u} - \underline{u}^*) + \frac{1}{2} (\underline{u} - \underline{u}^*)^T \underline{\nabla}_{\underline{u}}^2 G(\underline{u}^*) (\underline{u} - \underline{u}^*) = \underline{G}_2(\underline{u})$$

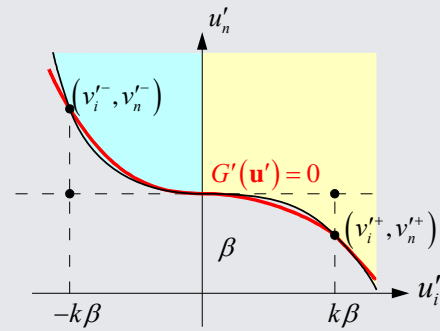
▲ **Calcul des courbures en P^* (UCB & IFMA)**

$$p_{f2} = \varphi(-\beta) \prod_{i=1}^{n-1} \frac{1}{\sqrt{1 + \psi(\beta) \kappa_i}}$$



▲ **Lissage par 1/2 paraboloïdes autour de P^* (UCB & IFMA)**

Der Kiureghian & De Stefano, 1991



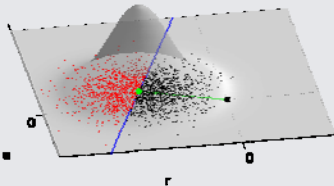
Monte Carlo simulation (MC) / Importance Sampling (IS)

▲ **MC (UCB & IFMA)**

$$p_f = \int_{D_{fX}} I(\underline{x}) f_X(\underline{x}) d\underline{x} \quad \blacktriangleright \quad p_f = E_{f_X} [I(\underline{X})]$$

▲ **IS (UCB & IFMA)**

$$p_f = \int_{D_h} I(\underline{x}) \frac{f_X(\underline{x})}{h(\underline{x})} h(\underline{x}) d\underline{x} \quad \blacktriangleright \quad p_f = E_h \left[I(\underline{X}) \frac{f_X(\underline{X})}{h(\underline{X})} \right]$$

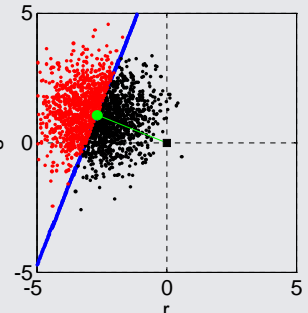
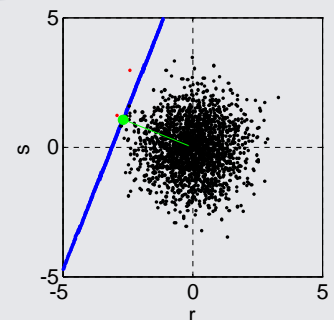


$$h(\underline{u}) = \frac{1}{(2\pi)^{n/2} \sqrt{\det \underline{\Sigma}}} \exp \left[-\frac{1}{2} (\underline{u} - \underline{u}^*)^T \underline{\Sigma}^{-1} (\underline{u} - \underline{u}^*) \right]$$

avec : $\underline{\Sigma} = \underline{I}$

$$\hat{p}_f = \frac{1}{N} \sum_{i=1}^N q_i$$

avec $q_i = I(\underline{x}_i)$ (MC) ou $q_i = I(\underline{x}_i) \frac{f_X(\underline{x}_i)}{h(\underline{x}_i)}$ (IS)



Structure analysisopt

```

% 'dspt': design point, 'origin': origin in standard normal space (simulation analysis)
analysisopt.sim_point = 'origin';

% 0: default rand matlab function, 1: Mersenne Twister (preferred)
analysisopt.rand_generator = 1;

% Standard deviation of sampling distribution in simulation analysis
analysisopt.stdv_sim = 1;

% Number of simulations in simulation analysis
analysisopt.num_sim = 1000000;

% Target coefficient of variation of failure probability in simulation analysis
analysisopt.target_cov = 0.01;
    
```



Directional Simulation (DS)

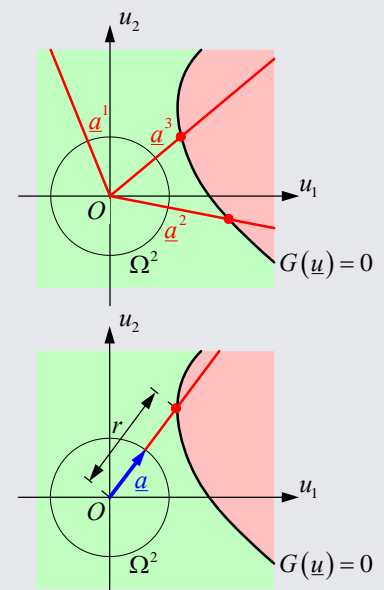
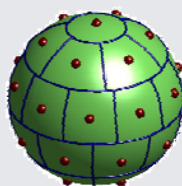
▲ Simulation directionnelle (IFMA)

Schueller & Stix, 1987

$$p_f = \int_{a \in \Omega^n} P[G(RA) \leq 0 \mid A = a] f_A(a) da$$

$$\hat{p}_f = \frac{1}{N} \sum_{i=1}^N [1 - \chi_n^2(r_i^2)]$$

Directions via points choisis de manière équirépartie sur l'hypersphère de rayon unité (IFMA)



Structure analysisopt

```

% Max search radius in standard normal space for directional simulation analysis
analysisopt.rho = 8;

% Tolerance for searching zeros of g function
analysisopt.tolx = 1e-5;

% Flag for storage of a values which gives axes along which simulations are carried out
analysisopt.keep_a = 1;

% Flag for storage of r values for which g(r) = 0
analysisopt.keep_r = 1;
    
```

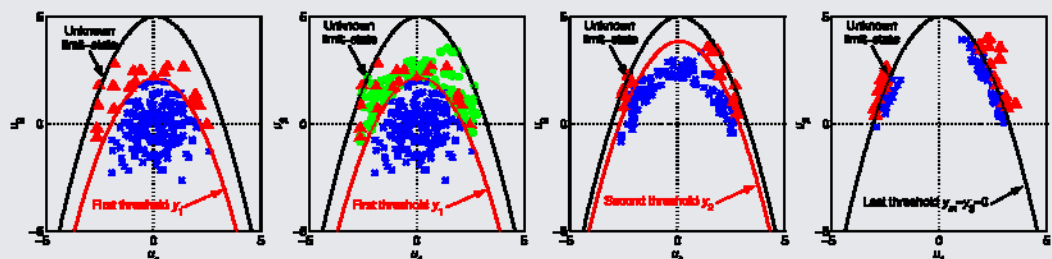
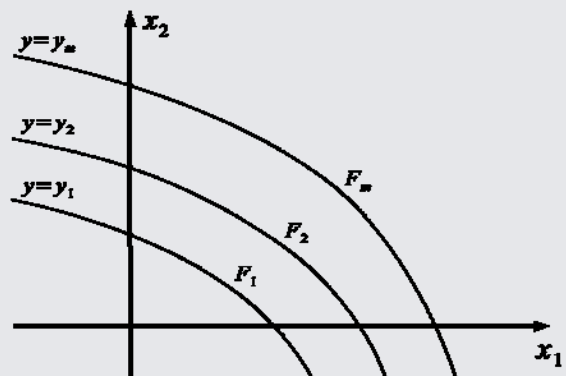


Subset Simulation (SS)

Subset simulation (IFMA)

Au & Beck, 2001

$$\begin{aligned}
 p_f &= P(F) \\
 &= P(F | F_{m-1}) P(F_{m-1}) \\
 &= \dots \\
 &= P(F_1) \prod_{i=2}^m P(F_i | F_{i-1})
 \end{aligned}$$



Structure analysisopt (suite)

```
analysisopt.width = 2;
```

```
analysisopt.pf_target = 0.1;
```

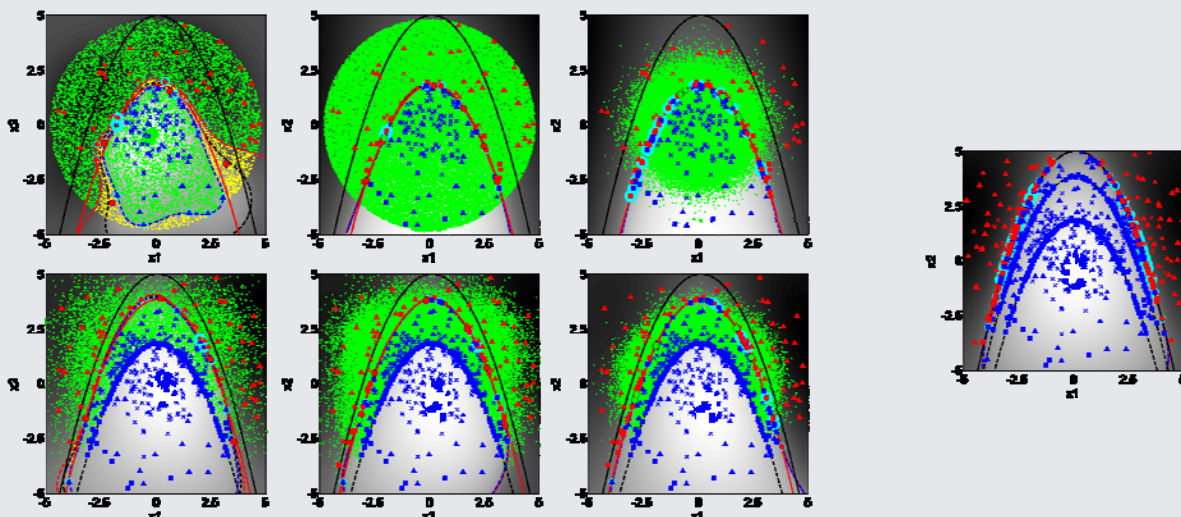
```
% i>=0 : restart from step i, -inf : full no save (default), -1 : full save  
analysisopt.ss_restart_from_step = -inf;
```



Subset Simulation and Support Vector Machines (²SMART)

▲ Subset simulation et SVMs (IFMA)

Deheeger, Bourinet & Lemaire, 2008



Structure analysisopt (suite)

```
analysisopt.num_sim_sdu = [ 50 50 ];
analysisopt.num_sim_norm = [ 100 100 ];
nrv = size(probdata.marg,1);
analysisopt.buf_size = round(1.25e7/nrv);
analysisopt.svm_buf_size = 3500^2;
analysisopt.flag_var_rbf = 0;
```



Facteurs d'importance

▲ Variables non-corrélées

$$\underline{\alpha} = - \frac{\nabla_{\underline{u}} G(\underline{u}^*)}{\|\nabla_{\underline{u}} G(\underline{u}^*)\|}$$

α - factors
Cosinus directeurs

▲ Variables corrélées

$$\underline{\gamma}^T = \frac{\underline{\alpha}^T \underline{J}_{\underline{u},x} \underline{\hat{D}}}{\|\underline{\alpha}^T \underline{J}_{\underline{u},x} \underline{\hat{D}}\|}$$

γ - factors

$\underline{\gamma}$ n'est en général pas un vecteur unitaire

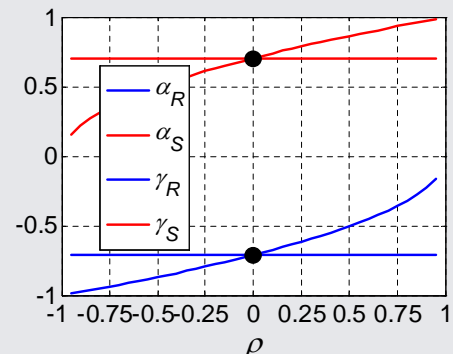
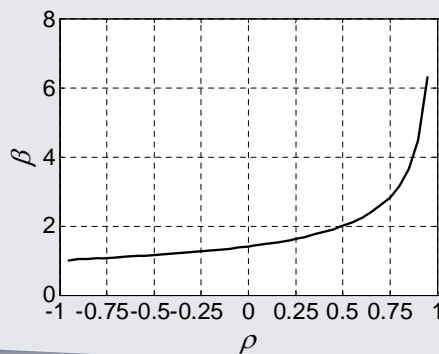
$\underline{\gamma} = \underline{\alpha}$ en l'absence de corrélation

$$R \sim N(4, 1)$$

$$S \sim N(2, 1)$$

$$\rho = [-0.95, 0.95]$$

$$g = r - s$$



Sensibilités fiabilistes

▲ Sensibilités fiabilistes

$$p_f = \int_{g(\underline{x}, \underline{\theta}_g) \leq 0} f_{\underline{x}}(\underline{x}, \underline{\theta}_f) d\underline{x} \quad \begin{array}{l} \underline{\theta}_f : \text{paramètres des lois - Ex. : } \mu, \sigma, p_1, p_2, \dots, \rho \\ \underline{\theta}_g : \text{paramètres (déterministes) de l'état-limite} \end{array}$$

- ↳ Sensibilités de β par rapport aux paramètres des lois

$$\nabla_{\underline{\theta}_f} \beta = \underline{\alpha}^T \underline{J}_{T, \underline{\theta}_f}(\underline{x}^*, \underline{\theta}_f)$$

$$\text{où } T : \underline{x} \rightarrow \underline{u} = T(\underline{x}, \underline{\theta}_f)$$

- ✓ Sensibilités de β par rapport aux paramètres de l'état-limite

$$\nabla_{\underline{\theta}_g} \beta = \frac{1}{\|\nabla_{\underline{u}} G(\underline{u}^*, \underline{\theta}_g)\|} \nabla_{\underline{\theta}_g} g(\underline{x}^*, \underline{\theta}_g)$$

- ✓ Sensibilités de p_f par rapport aux paramètres des lois / de l'état-limite

$$\begin{aligned} \nabla_{\underline{\theta}_f \text{ ou } \underline{\theta}_g} p_{f1} &= -\varphi(-\beta) \nabla_{\underline{\theta}_f \text{ ou } \underline{\theta}_g} \beta \\ &= -\varphi(\beta) \nabla_{\underline{\theta}_f \text{ ou } \underline{\theta}_g} \beta \end{aligned}$$



Sensibilités fiabilistes (suite)

▲ Sensibilités aux paramètres des distributions (UCB & IFMA)

- ↳ calcul approché (UCB)
- ↳ Calcul "exact" par intégration numérique (IFMA)

$$\frac{\partial u}{\partial \theta_f} = \underline{L}_0^{-1} \frac{\partial z}{\partial \theta_f} + \frac{\partial \underline{L}_0^{-1}}{\partial \theta_f} z$$

$$\begin{aligned} \theta_f &= \mu_i, \sigma_i \\ &= p_{1i}, p_{2i}, \dots \end{aligned}$$

Classiquement pris en compte

Généralement négligé

▲ Sensibilités à la corrélation entre variables aléatoires (IFMA)

$$\frac{\partial u_k}{\partial \rho_{ij}} = \sum_{l=1}^k \frac{\partial l_{0,kl}^{-1}}{\partial \rho_{ij}} \Phi^{-1} [F_{X_l}(x_l)]$$

$$\theta_f = \rho_{ij}$$



Structures probdata, analysisopt et gfundata

```
probdata.flag_sens = 1;
```

$$\nabla_{\theta_f} \beta, \nabla_{\theta_f} P_{f1}$$

```
% Parameter for computation of FFD estimates of dbeta_dthetag
% Perturbation = thetag/analysisopt.ffdpara_thetag if thetag ~= 0
%               or 1/analysisopt.ffdpara_thetag if thetag == 0;
% Recommended values: 1000 for basic limit-state functions,
%                   100 for FE-based limit-state functions
analysisopt.ffdpara_thetag = 1000;
```

$$\nabla_{\theta_g} \beta, \nabla_{\theta_g} P_{f1}$$

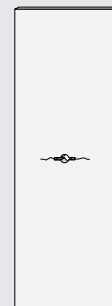
```
gfundata(1).flag_sens = 0;
```



Sensibilité à la corrélation

▲ Geometry ► Center-cracked tension specimen M(T)

$W = 6''$: width $\sigma = 60.45$ MPa : max applied stress
 $e = 0.1''$: thickness $R = 0.2$: stress ratio



▲ Limit-state function

$$g(C, m) = N_R(C, m) - N_S$$

where

$$N_R = \int_{a_i}^{a_f} \frac{1}{C \Delta K^m} da$$

N_S : target service life

with a_i : initial crack half-length
 $a_f = 49.8$ mm : final crack half-length

$$\frac{da}{dN} = C \Delta K^m \quad (\text{Paris-Erdogan law})$$

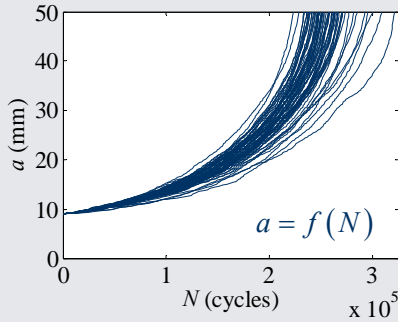
$$\Delta K = K_{\max} (1 - R) \quad \text{and} \quad K_{\max} = \frac{1 - 0.025(a/W)^2 + 0.06(a/W)^4}{\sqrt{\cos(\pi a/W)}} \sigma \sqrt{\pi a}$$



Sensibilité à la corrélation

▲ Stochastic model

Statistics of C and m
from Virkler's data
(68 samples)



Virkler et al., 1979

↳ Case #1

Variable	Type	Mean	Stdv.	Correlation
a_i	-	4.4	-	-
$\ln C$	normal	-26.056	0.972	-0.99795
m	normal	2.855	0.166	-

↳ Case #2

Variable	Type	Mean	Stdv.	Correlation
a_i	exp.	1.5	1.5	-
$\ln C$	normal	-26.056	0.972	-0.99795
m	normal	2.855	0.166	-



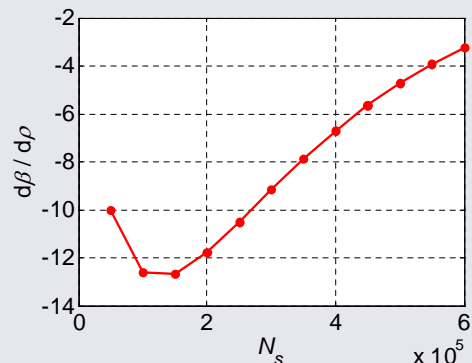
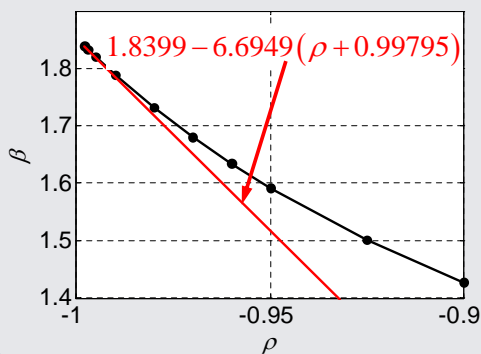
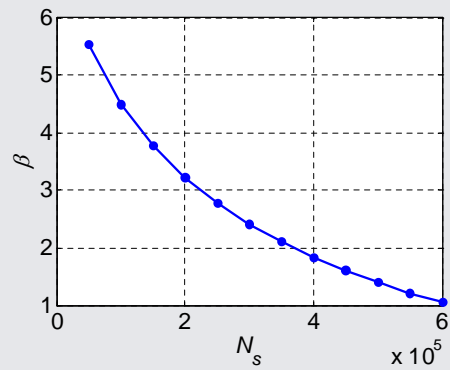
Sensibilité à la corrélation

↳ Case #1 $N_s = 400000$ cycles

$$\beta = 1.8005 \quad \frac{d\beta}{d\rho} = -368.39$$

↳ Case #2 $N_s = 400000$ cycles

$$\beta = 1.8399 \quad \frac{d\beta}{d\rho} = -6.6949$$



▲ Subset simulation et SVMs (IFMA)

Royset & Polak, 2004 (RBDO basée sur MC and IS)

Royset & Polak, 2007 (RBDO basée sur DS)

Nested bi-Level Approach (N2LA)

- ↳ Méthode d'optimisation fiabiliste basée sur gradients
- ↳ Méthode fiabiliste appelée à l'intérieur de la boucle d'optimisation
- ↳ Algorithme d'optimisation de Polak-He (Polak, 1997)
- ↳ Recherche de la direction de descente basée sur l'algorithme SQP
- ↳ Pas dans la direction de descente suivant règle d'Armijo

Travaux IFMA

- ↳ Résolution de la dépendance aux ordres de grandeur des fonctions coût, contraintes et variables d'optimisation
- ↳ Implémentation de FORM, FORM système, MC, IS, DS
- ↳ Vectorisation des algorithmes (partiel)



Structures rbdo_fundata et rbdo_parameters

```
rbdo_fundata.cost = {
    '4/3.*pi.*(r1.^3 - r0.^3)'
    '0e2 * 4/3.*pi.*(r1.^3 - r0.^3)' };

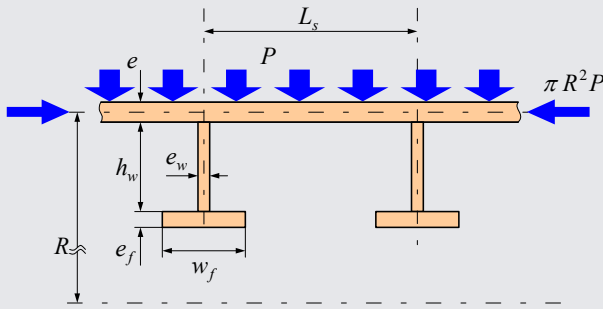
rbdo_fundata.constraint = { '- r0 + 40'
    '- r1'
    'r0 - 150'
    'r1 - 200000'
    'r0 - r1'};

rbdo_parameters.alpha = 0.5;
rbdo_parameters.beta = 0.6;
rbdo_parameters.gamma = 1;
rbdo_parameters.delta = 1;
rbdo_parameters.tau = 0.9999;
rbdo_parameters.eta = 0.0001;

rbdo_parameters.max_iter = 10;
rbdo_parameters.Nmax = 1e8;
rbdo_parameters.steplim = 20;
rbdo_parameters.target_improvement_ratio = 0; % * initial_cost_value
rbdo_parameters.target_beta = 4.5;
% Underlying reliability method: 'FORM' or 'FORM_Serials' or 'MC' or 'dirsim'
rbdo_parameters.method = 'FORM';
```

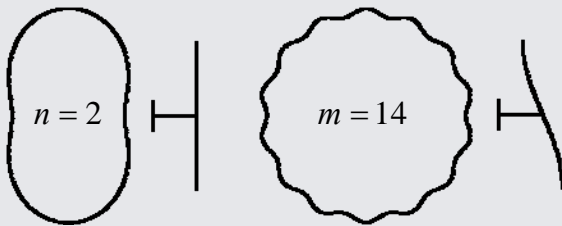


Exemple RBDO - Collapse plastique d'une coque raidie



Imperfections prise en compte

$$A(\theta, z) = A_n \cos n\theta + A_m \cos \frac{\pi z}{L_s} \cos m\theta$$



Variables aléatoires

Variable	Distribution	Moyenne	C.d.v.
A_{n2}	Lognormal		50%
A_{m14}	Lognormal		50%
E	Lognormal	200 GPa	3%
σ_y	Lognormal	390 MPa	5%

Variables d'optimisation déterministes

R	2488	● h_w	156
L_s	600	● e_w	10
● e	24	● w_f	120
		● e_f	24

Fonction coût

M (poids de la coques) / V (déplacement)

Contraintes déterministes

Critères de déversement de raidisseur (BS 5500)



Exemple RBDO

`rbdo_fundata.cost =`

```
{ '7850 * ( ((2488+e/2)^2-(2488-e/2)^2)*600 + ((2488-e/2)^2-(2488-(e/2+hw))^2)*ew + ((2488-(e/2+hw))^2-(2488-(e/2+hw+ef))^2)*wf ) / ( 1026 * (2488+e/2)^2 * 600 )'
```

```
'0e2 * 7850 * ( ((2488+e/2)^2-(2488-e/2)^2)*600 + ((2488-e/2)^2-(2488-(e/2+hw))^2)*ew + ((2488-(e/2+hw))^2-(2488-(e/2+hw+ef))^2)*wf ) / ( 1026 * (2488+e/2)^2 * 600 )' };
```

```
rbdo_fundata.constraint = { ' -e + 10 '
' -hw + 75 '
' -ew + 5 '
' -wf + 50 '
' wf - 175 '
' -ef + 5 '
' hw - (1.1*(2e5/390)^0.5) * ew '
' wf/2 - (0.5*(2e5/390)^0.5) * ef '
' hw - 2*wf '
' wf - hw ' };
```



Distribution de FERUM

▲ Planning prévisionnel de mise en ligne

Fin 2008

A faire :

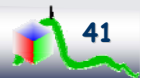
- ↳ Rédaction d'un guide utilisateur, d'un recueil d'exemples
- ↳ Site web (IFMA et miroir UCB)
- ↳ ... un peu d'ordre à apporter et des commentaires à ajouter

▲ Philosophie

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Vous êtes libre d'utiliser ... mais vos contributions sont les bienvenues 😊



OpenSees

▲ Site internet

<http://opensees.berkeley.edu/>

▲ Document de référence (partie fiabiliste)

http://www.inrisk.ubc.ca/opensees/ubc_opensees_guide.pdf

